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Radiation reaction and mass renormalization in scalar and tensor fields and linearized gravitation

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Abstract. The analytic continuation of the equation of motion provides a simple method to obtain the radiation reaction force and mass renormalization in classical field theories as an alternative to the energy–momentum tensor method. The coefficient of the Abraham factor is $+\frac{1}{3}$, $\frac{2}{3}$, $-\frac{5}{3}$ and $-\frac{11}{3}$ for the scalar, vector, tensor fields, and linearized gravitation, respectively.

1. Introduction

The standard method of deriving the classical covariant equations of motion of point particles including radiation reaction is via the energy–momentum tensor $T_{\mu\nu}$ of the field (Dirac 1938). In its most satisfactory form (Teitelboim 1970, Villarroel 1974), it consists of dividing $T_{\mu\nu}$ into two parts, $T_{\mu\nu}^{(b)}$ and $T_{\mu\nu}^{(r)}$. The energy–momentum $P_{\mu}^{(r)}$ corresponding to the pure radiation part $T_{\mu\nu}^{(r)}$ gives the Larmor radiation term in the equation of motion, $\dot{P}_{\mu}^{(r)} = (2e^2/3)(\ddot{z}^2\dot{z}_{\mu})$. The momentum $P_{\mu}^{(b)}$ corresponding to the energy–momentum tensor $T_{\mu\nu}^{(b)}$ bound around the particle, after integration over a tube along the world line and a limiting procedure, gives the mass renormalization (δm), and the Schott term, $\dot{P}_{\mu}^{(b)} = (m+\delta m)\ddot{z}_{\mu} - \frac{2}{3}e^2\dddot{z}_{\mu}$, and the sum of the two momenta the desired equation of the motion. This procedure thus uses the energy–momentum properties of the field alone.

An alternative procedure which uses the particle properties alone has been recently given (Barut 1974) and turned out to be extremely simple. It consists of an analytic continuation procedure of the whole equation of the motion, $m\ddot{z}_{\mu} = eF_{\mu\nu}^{\mathrm{ret}}\dot{z}_{\nu} + K_{\mu}$, to the world line.

In this paper we show how this method of analytic continuation works with equal simplicity in the case of scalar and tensor fields.

2. Scalar field

The equation of motion of a point particle in a scalar field $\phi(x)$ (Barut 1964, p 56)

$$\frac{\mathrm{d}}{\mathrm{d}s}[(m+g\phi)\dot{z}_{\mu}] = g\phi_{,\mu}, \tag{1}$$

or,

$$m\ddot{z}_\mu + g\phi\ddot{z}_\mu + g\phi_{,\nu}\dot{z}^\nu\dot{z}_\mu = g\phi_{,\mu}. \tag{1'}$$

The retarded field at the point x produced by the particle at the retarded point $z(s)$ is:

$$\phi(x, z(s)) = -g/R, \tag{2}$$

where g is the coupling constant and

$$R = (x-z(s))^\mu \dot{z}_\mu(s). \tag{3}$$

The method consists in considering $\phi(x, z(s))$ and $\phi_{,\mu}$ as a function of two variables x and $z(s)$ and continuing equation (1) to the unphysical point $x = z(s+u)$, keeping the second variable fixed at $z(s)$ (Barut 1974):

$$\frac{\mathrm{d}}{\mathrm{d}s}\{[m+g\phi(x = z(s+u), z(s))]\dot{z}_\mu(s+u)\} = g\phi_{,\mu}(x = z(s+u), z(s)). \tag{4}$$

From (2) we have further

$$\phi_{,\mu} = \frac{g}{R^2}[\dot{z}_\mu - (x-z)_\mu(1-Q)/R], \tag{5}$$

where

$$Q = (x-z(s))^\mu \ddot{z}_\mu(s). \tag{6}$$

We now insert (2) and (5) into (4), use the expansions, at $x = z(s+u)$,

$$\begin{aligned} R &= u + \tfrac{1}{6}u^3\dot{z}\dddot{z} + \dots \\ Q &= \tfrac{1}{2}u^2\ddot{z}^2 + \tfrac{1}{6}u^3\ddot{z}\dddot{z} + \dots \\ (x-z) &= u\dot{z} + \tfrac{1}{2}u^2\ddot{z} + \tfrac{1}{6}u^3\dddot{z} + \dots, \end{aligned} \tag{7}$$

where the quantities on the right-hand side all refer to the point s.

Equation (4) then becomes

$$m\ddot{z}_\mu(s+u) - \ddot{z}_\mu(s+u)\left(\frac{g^2}{u} + \mathrm{O}(u^2)\right) - \dot{z}_\mu(s+u)\dot{z}^\nu(s+u)\left(\frac{g^2}{2u}\ddot{z}_\nu + \frac{g^2}{6}\dddot{z}_\nu - \frac{g^2}{3}\ddot{z}^2\dot{z}_\nu + \mathrm{O}(u^2)\right)$$

$$= -\left(\frac{g^2}{2u}\ddot{z}_\mu + \frac{g^2}{6}\dddot{z}_\mu - \frac{g^2}{3}\ddot{z}^2\dot{z}_\mu + \mathrm{O}(u^2)\right).$$

In the right-hand side we write $\ddot{z}_\mu = \ddot{z}_\mu(s+u) - u\dddot{z}_\mu + \mathrm{O}(u^2)$:

$$\ddot{z}_\mu(s+u)\left(m - \frac{g^2}{u}\right) - \dot{z}_\mu(s+u)\left[\left(\frac{g^2}{6}\right)\dot{z}\dddot{z} + \left(\frac{g^2}{6}\right)\ddot{z}^2\right]$$

$$= -\frac{g^2}{2u}\ddot{z}_\mu(s+u) + \frac{g^2}{2}\dddot{z}_\mu - \frac{g^2}{6}\dddot{z}_\mu + \frac{g^2}{3}\ddot{z}^2\dot{z}_\mu + \mathrm{O}(u),$$

or, finally

$$\ddot{z}_\mu(s+u)\left(m - \frac{g^2}{2u}\right) = \frac{g^2}{3}(\dddot{z}_\mu + \dot{z}_\mu\ddot{z}^2) + \mathrm{O}(u). \tag{8}$$

Now we first perform mass renormalization by putting $m_{\mathrm{exp}} = m - g/2u$ and then go to the limit $u \to 0$ and obtain:

$$m_{\mathrm{exp}}\ddot{z}_\mu = \frac{g^2}{3}(\dddot{z}_\mu + \dot{z}_\mu\ddot{z}^2). \tag{9}$$

In addition of course, the external force K_μ should be added to the right-hand side of equation (9).

3. Tensor field

The equation of motion of a point particle in a tensor field $\psi_{\sigma\mu}$ is (Barut 1964 p 63)

$$\frac{m}{g}\ddot{z}_\mu = \frac{\mathrm{d}}{\mathrm{d}s}[(\psi_{\mu\nu}+\psi_{\nu\mu})\dot{z}^\nu - \psi_{\sigma\rho}\dot{z}^\sigma\dot{z}^\rho\dot{z}_\mu] - \psi_{\sigma\rho,\mu}\dot{z}^\sigma\dot{z}^\rho. \tag{10}$$

The retarded field, the counterpart of (2), is

$$\psi_{\mu\nu} = g\dot{z}_\mu\dot{z}_\nu/R, \tag{11}$$

with R as given in (3). The field derivatives are

$$\psi_{\mu\nu,\lambda} = g[\ddot{z}_\mu\dot{z}_\nu(x-z)_\lambda R^{-2} - \dot{z}_\mu\dot{z}_\nu\dot{z}_\lambda R^{-2} + \dot{z}_\mu\dot{z}_\nu(x-z)_\lambda R^{-3}(1-Q) + (\mu\leftrightarrow\nu)]. \tag{12}$$

Exactly the same expansions (7) and the same procedures as before yield

$$\left(m-\frac{g^2}{2u}\right)\ddot{z}_\mu(s+u) = -\tfrac{5}{3}g^2(\dddot{z}_\mu+\ddot{z}^2\dot{z}_\mu)+\mathrm{O}(u), \tag{13}$$

or, after mass renormalization and in limit $u\to 0$,

$$m_{\exp}\ddot{z}_\mu = -\tfrac{5}{3}g^2(\dddot{z}_\mu+\ddot{z}^2\dot{z}_\mu). \tag{14}$$

We note that the tensor field

$$\varphi_{\mu\nu} = -gg_{\mu\nu}/R \tag{15}$$

is also a solution of the field equations and gives, up to a sign, the same Larmor factor as the scalar case (9). Equation (15) is, up to a factor, the trace of the field (11).

The linearized gravitation equations are equivalent to a tensor field and a scalar field. The tensor field comes with a factor 2 relative to the scalar field (see, for example, Weber 1961 p 95). Thus the factor in front of the Abraham force is

$$2(-\tfrac{5}{3})-\tfrac{1}{3} = -\tfrac{11}{3},$$

Table 1. Radiation reaction force and mass renormalization terms

Massless fields	Retarded field ($R=(x-z)\dot{z}$)	Abraham factor ($\Gamma_\mu = \dddot{z}_\mu+\ddot{z}^2\dot{z}_\mu$)	Mass renormalization δm
Scalar	$-gR^{-1}$	$+\frac{1}{3}\Gamma_\mu g^2$	$+\frac{g^2}{2u}$
Vector	$\left(\frac{e}{4\pi\epsilon_0}\right)\dot{z}_\mu R^{-1}$	$+\frac{2}{3}\Gamma_\mu\left(\frac{e^2}{4\pi\epsilon_0}\right)$	$\frac{1}{2u}\left(\frac{e^2}{4\pi\epsilon_0}\right)$
Tensor	$g\dot{z}_\mu\dot{z}_\nu R^{-1}$ $-gg_{\mu\nu}R^{-1}$	$-\frac{5}{3}\Gamma_\mu g^2$ $-\frac{1}{3}\Gamma_\mu g^2$	$-g^2/2u$
Linearized gravity	$g(2\dot{z}_\mu\dot{z}_\nu-g_{\mu\nu})R^{-1}$	$-\frac{11}{3}\Gamma_\mu g^2$	

a result which has been obtained by other authors before (Havas and Goldberg 1962, Smith and Havas 1965). Table 1 summarizes the radiation reaction force and mass renormalization terms in various cases derived by the analytic continuation method.

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